

EPDs and RISK

6-16-2016 DVV

EPDs and RISK
What is accuracy?
Do we need accuracy?

6-16-2016 DVV

TOPICS

Emphasis on accuracy (a limited survey)

History

- Accuracy
- Reliability
- BIF-Accuracy
- Future Progeny Accuracy

Standard Error of Prediction

- Confidence Ranges
- The Gambler

TOPICS (cont'd)

Standard Error of Prediction

- Difference between pair of EBV
- Covariance between a pair of EBV
- One table for all traits?

G-BLUP

Bayes-Gibb's sampling chains

EBV; EPD = EBV/2 *****

Good luck!

A SURVEY

Bull	EBV	Accuracy
X	+10.0	0.60
Y	+10.0	0.90

Choose: X or Y ?

Bull	EBV	Accuracy
X	+10.0	0.60
XX	+ 4.0	0.90

Choose: X or XX ?

Bull	EBV	Accuracy ^{REL}
X	+10.0	0.36
XX	+ 4.0	0.81

Choose: X or XX ?

Bull	EBV	Accuracy ^{BIF}
X	+10.0	0.20
XX	+ 4.0	0.56

Choose: X or XX ?

Bull	EBV	Accuracy ^{PROG}
X	+10.0	0.30
XX	+ 4.0	0.45

Choose: X or XX ?

HISTORY

Traditionally: **Accuracy = Correlation**
 Correlation(EPD, true PD) [0 <-> 1]
 Correlation(EBV, true BV) [0 <-> 1]
Squared-Accuracy is fraction of genetic variance accounted for by EBV [0 <-> 1]

A CONFLICT?

EPD = EBV/2 has dollar value.
 High accuracy is reassuring.
 Too much emphasis on accuracy?
 Not directly associated with \$\$\$
 How to de-emphasize accuracy?
 The *Dairy* attempt
 The *Beef* attempt

RELIABILITY

The *DAIRY* attempt (successful?)
 Predicted Transmitting Ability
 PTA = EPD = EBV/2
Reliability = Squared-Accuracy
 Fraction of genetic variance accounted for by EBV
 Limits are 0.0 <-> 1.0
 Paul VanRaden ~ 1989

<u>Accuracy</u>	<u>Reliability</u>
0.20	0.04
0.30	0.09
0.40	0.16
0.50	0.25
0.60	0.36
0.70	0.49
0.80	0.64
0.90	0.81
0.99	0.98

BIF-ACCURACY
 The *BEEF* attempt (successful, confusing?)
BIF-Accuracy
 Birthed near a tornado in Atlanta in 1984!
 Uses squared-accuracy
 Tracks Standard Error of Prediction (SEP)
 $SEP = \text{SQRT}[\text{Variance of Prediction Error}]$
 Need Variance(breeding values) = V_g
 Richard Willham ~ 1984

Prediction Error, PE
 $PE = (EBV - \text{True BV})$
 Variance of PE = $\text{Var}[EBV - \text{True BV}]$
 $\text{Var}(PE) = (1 - \text{squared-accuracy}) \times \text{Var}(BV)$
 $= (1 - acc^2)V_g$
Standard Error of Prediction
 $SEP = \text{SQRT}[\text{Var}(PE)] = \text{SQRT}[(1 - acc^2)V_g]$

BIF-Accuracy
 $BIF\text{-Accuracy} = 1 - \text{SQRT}[(1 - acc^2)]$
 $\text{SQRT}[(1 - acc^2)]$ is standardized SEP
 $SEP = \text{SQRT}[(1 - acc^2)V_g]$
 As acc^2 increases towards ONE,
 SEP decreases towards ZERO
 As SEP decreases towards ZERO,
 BIF-accuracy goes to ONE
 Limits are $0 \leftrightarrow 1.0$

<u>Accuracy</u>	<u>Reliability</u>	<u>BIF-Accuracy</u>
0.20	0.04	0.02
0.30	0.09	0.05
0.40	0.16	0.08
0.50	0.25	0.13
0.60	0.36	0.20
0.70	0.49	0.28
0.80	0.64	0.40
0.90	0.81	0.56
0.99	0.98	0.86

Future Progeny Accuracy
 Progeny w/records \gg Bull $EPD = EBV/2$
 EPD is expected transmission to future progeny
Accuracy for any new progeny is one-half accuracy of EPD or EBV of the sire*
 Progeny accuracy can never exceed 0.50 unless records of self, dam, etc.

Accuracy	Reliability	BIF Acc	Prog Acc
0.30	0.09	0.05	0.15
0.40	0.16	0.08	0.20
0.50	0.25	0.13	0.25
0.60	0.36	0.20	0.30
0.70	0.49	0.28	0.35
0.80	0.64	0.40	0.40
0.90	0.81	0.56	0.45
0.99	0.98	0.86	0.50

Summary so far

Four Measures of Accuracy

Bull	EBV	Acc ¹	Acc ^R	Acc ^B	Acc ^P
X	+10.0	0.60	0.36	0.20	0.30
XX	+ 4.0	0.90	0.81	0.56	0.45

Choose: X or XX ?

EPD = EBV/2 has \$\$\$ value.

What is \$\$\$ value of RISK (accuracy)?

Need quantitative measure of RISK

Names for Quantitative Measures of Risk

- Margin of Error
- Possible Change
- Confidence Range
- Standard Deviation
- Standard Error of Prediction

In units of measurement (e.g., lb)

Accuracies are unit-less (zero to one)

Usual Quantitative Measure of Risk

Standard Error of Prediction*

- Possible (Probable) Change
- Confidence Range

Uses traditional accuracy (squared)

Uses genetic standard deviation, SQRT[Vg]

Confidence Range assumes 'normality', the bell-shaped curve

Review Standard Error of Prediction

PE = (EBV – True BV)

Variance of PE = Var[EBV – True BV]

Var(PE) = (1 – squared-accuracy) x Var(BV)

= (1 – acc²)Vg(lb²)

Standard Error of Prediction

SEP = SQRT[Var(PE)]

= SQRT[(1 – acc²)Vg](lb)

As acc² increases, SEP decreases towards ZERO, dependent on SQRT[(1 – acc²)]

Accuracy	Reliability	BIF Acc	SEP(lb)
0.30	0.09	0.05	22.8
0.40	0.16	0.08	21.0
0.50	0.25	0.13	18.8
0.60	0.36	0.20	16.0
0.70	0.49	0.28	12.8
0.80	0.64	0.40	9.0
0.90	0.81	0.56	4.8
1.00	1.00	1.00	0.0

Accuracy	Reliability	BIF Acc	SEP(lb)	p
0.30	0.09	0.05	22.8	1.5
0.40	0.16	0.08	21.0	2.9
0.50	0.25	0.13	18.8	5.0
0.60	0.36	0.20	16.0	8.4
0.70	0.49	0.28	12.8	14.4
0.80	0.64	0.40	9.0	26.7
0.90	0.81	0.56	4.8	64.0
1.00	1.00	1.00	0.0	++++

CONFIDENCE RANGES
 $SEP = \text{SQRT}[\text{Var}(PE)] = \text{SQRT}[(1 - \text{acc}^2)Vg]$
 Is a standard measure of risk
68% Confidence Range to include true BV
 From $[EBV - (1)SEP]$ to $[EBV + (1)SEP]$
 Chance BV **greater than** $[EBV + (1)SEP]$, **16%**
 Chance BV **less than** $[EBV - (1)SEP]$ is **16%**
 Chance $BV > EBV = 50\%$
 Chance $BV < EBV = 50\%$

95% Confidence Range to include true BV
 From $[EBV - (2)SEP]$ to $[EBV + (2)SEP]$
 Chance $BV >>> [EBV + (2)SEP]$ is **2.5%**
 Chance true $BV <<< [EBV - (2)SEP]$ is **2.5%**
 Range not limited to (1) or (2) SEP
50% Confidence Range:
 $SEP \times (0.7)$ gives about 25%,25%,25%,25%
 Equal chance of going up or going down!

Bulls: X and Y
 $SEP = \text{SQRT}[(1 - \text{acc}^2) \times Vg]$
 $Vg = 625(\text{lb})^2$ $\text{SQRT}[Vg] = 25(\text{lb})$
 Bull X, **accuracy = 0.60**
 $SEP(X) = \text{SQRT}[(1 - 0.36) \times 625] = 20.0(\text{lb})$
 Bull Y, **accuracy = 0.90**
 $SEP(Y) = \text{SQRT}[(1 - 0.81) \times 625] = 10.9(\text{lb})$

Bull	EBV	Accuracy	SEP
X	+10.0	0.60	20.0
Y	+10.0	0.90	10.9

68% Confidence Ranges:
 Bull X: $10.0 - (1)(20.0)$ to $10.0 + (1)(20.0)$
 [-10.0 to +30.0]
 Bull Y: $10.0 - (1)(10.9)$ to $10.0 + (1)(10.9)$
 [-0.9 to +20.9]
 Choose: X or Y ?

Bull	EBV	Accuracy	SEP
X	+10.0	0.60	20.0
Y	+10.0	0.90	10.9

95% Confidence Ranges:
 Bull X: $10.0 - (2)(20.0)$ to $10.0 + (2)(20.0)$
 [-30.0 to +50.0]
 Bull Y: $10.0 - (2)(10.9)$ to $10.0 + (2)(10.9)$
 [-11.8 to +31.8]
 Choose: X or Y ?

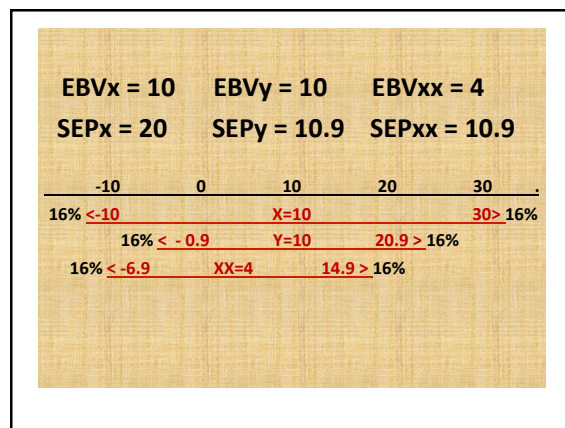
The Gambler:
 Bull X (accuracy = 0.60),
 16% > 30.0
 2.5% > 50.0
 Bull Y (accuracy = 0.90),
 16% > 20.9
 2.5% > 31.8
But, as many down, as up!

Bull	EBV	Accuracy	SEP
X	+10.0	0.60	20.0
XX	+ 4.0	0.90	10.9

68% Confidence Ranges:
 Bull X: [10.0 – (1)(20.0)] to [10.0+(1)(20.0)]
 [-10.0 to +30.0]
 Bull XX: [4.0 – (1)(10.9)] to [4.0 +(1)(10.9)]
 [-6.9 to +14.9]
 Choose: X or XX ?

Bull	EBV	Accuracy	SEP
X	+10.0	0.60	20.0
XX	+4.0	0.90	10.9

95% Confidence Ranges:
 Bull X: [10.0 – (2)(20.0)] to [10.0+(2)(20.0)]
 [-30.0 to +50.0]
 Bull XX: [4.0 – (2)(10.9)] to [4.0+(2)(10.9)]
 [-17.8 to +25.8]
 Choose: X or XX ?



Can extend to comparison of two animals:
 Prediction of Difference: EBV(X) – EBV(Y)
 Accuracy of difference between X and Y?
 Now SE of Prediction of Difference (SEPD)
 rather than SEP
Is not SE of an EPD!
SEP(EPD_x)= SEP(EPD_x – Constant)
 Now confidence ranges are centered
 on difference: EBV(X) – EBV(Y)

Prediction Error for Difference (PED):
 PED(X-Y) = [(EBV_x – BV_x) – [(EBV_y – BV_y)]
 PED = PE_x – PE_y
 Variance(PED) = Variance(PE_x – PE_y)
 [review: (A – B)² = A² + B² – 2(AB)]
 Variance(PED) = V(PE_x) + V(PE_y)
 – (2)Covariance(PE_x, PE_y)
 V(PED) = [(1 – acc_x²)Vg + (1 – acc_y²)Vg]
 – (2)Covariance(PE_x, PE_y)

Prediction Error for Difference in EBV:
 $V(PED) = [(1 - accx^2) + (1 - accy^2)]Vg - (2)Covariance(PE_x, PE_y)$
Covariance(PE_x, PE_y):
 Likely small vs. V(PE_x) and V(PE_y) except close relatives (sire-son, PHS)
 Difficult to obtain with many animals
 V(PE_x) and V(PE_y) usually **approximated** from **approximated** accx² and accy²
 Not easy to approximate Cov(PE_x, PE_y)

$V(PED) = [(1 - accx^2) + (1 - accy^2)]Vg - (2)Covariance(PE_x, PE_y)$
If ignore Cov(PE_x, PE_y), say it is ZERO, then
 $V(PED) = [2 - accx^2 - accy^2]Vg$ and standard error of EBV_x - EBV_y is (SEPD_{x-y})
 $SEPD^* = SQRT[(2 - accx^2 - accy^2)Vg]$
 SEPD* somewhat greater than SEPD
 Works for any trait, need only its Vg
 Makes easy to apply?

$V(PED) = [(1 - accx^2) + (1 - accy^2)]Vg - (2)Covariance(PE_x, PE_y)$

V(PE _x)	V(PE _y)	r	C(PE _x , PE _y)	SEPD
100	100	0.00	0	14.14
100	100	0.05	5	13.78
100	100	0.10	10	13.42
400	100	0.00	0	22.37
400	100	0.05	10	21.97
400	100	0.10	20	21.25

Prediction error for difference in EBV:
 $SEPD = SQRT[(2 - accx^2 - accy^2)Vg]$
 Works for **ANY TRAIT or INDEX**, need only the Vg for the trait or INDEX
 Many possible pairs of EBV:
 most not of interest
 1) For specific pair, 'hand' calculator
 2) Use a table for pairs of accuracy, then one multiplication

Prediction error for difference in EBV:
 $SEPD = SQRT[(2 - accx^2 - accy^2)Vg]$
 Table for pairs of acc: accx and accy

	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
0.05										
0.15										
0.25		1.41								
0.35			SQRT[2.0 - accx² - accy²]							
0.45		1.40	1.39							
0.55				1.38						
0.65					1.37					
						1.30				
							1.28			
								1.07		

Prediction error for difference in EBV:
 $SEPD = SQRT[(2 - acc1^2 - acc2^2)Vg]$
 A table for pairs of acc: acc1 and acc2

	For our example bulls			
.acc1/acc2	0.30	0.60	0.90	.
0.30	1.35	1.25	1.05	
0.60	1.25	1.13	0.91	
0.90	1.05	0.91	0.62	

A table for pairs of acc: accx and accy

.	0.30	0.60	0.90	.
0.30	1.35	1.25	1.05	
0.60	1.25	1.13	0.91	
0.90	1.05	0.91	0.62	

For Bull X (acc = 0.60)
 For Bull Y (or XX) (acc = 0.90)
 Table value for (X,Y) = **0.91**;
 With $V_g = 625$ and $\text{SQRT}(625) = 25$
 $\text{SEPD}(X - Y) = 0.91 \times 25 = 22.75$

A table for pairs of acc: accxx and accy

.	0.30	0.60	0.90	.
0.30	1.35	1.25	1.05	
	0.60	1.25	1.13	0.91
0.90	1.05	0.91	0.62	

For bull Y (acc = 0.90)
 For bull XX (acc = 0.90)
 Table value (Y,XX) = **0.62**
 With $V_g = 625$ and $\text{SQRT}(625) = 25$
 $\text{SEPD}(Y - XX) = 0.62 \times 25 = 15.50$

With $V_g = 625$ and $\text{SQRT}(625) = 25$
 $\text{SEPD}(X - Y) = 0.91 \times 25 = 22.75$
 $\text{Sepd}(X - XX) = 0.91 \times 25 = 22.75$
 $\text{SEPD}(Y - XX) = 0.62 \times 25 = 15.50$

“t-value’s”

$\text{EBV}(X) - \text{EBV}(Y): [10 - 10]/22.75 = 0.00$
 $\text{EBV}(X) - \text{EBV}(XX): [10 - 4]/22.75 = 0.26$
 $\text{EBV}(Y) - \text{EBV}(XX): [10 - 4]/15.50 = 0.39$

$\text{SEPD}(X-Y), \text{SEPD}(X-XX) = 0.91 \times 25 = 22.75$
 $\text{SEPD}(Y-XX) = 0.62 \times 25 = 15.50$

68% Confidence Ranges

$\text{EBV}(X) - \text{EBV}(Y): [0 - 22.75] \text{ to } [0 + 22.75]$
 $[10 - 10]: \quad [-22.75 \text{ to } +22.75]$
 $\text{EBV}(X) - \text{EBV}(XX): [6 - 22.75] \text{ to } [6 + 22.75]$
 $[10 - 4]: \quad [-16.75 \text{ to } +28.75]$
 $\text{EBV}(Y) - \text{EBV}(XX): [6 - 15.50] \text{ to } [6 + 15.50]$
 $[10 - 4]: \quad [-9.50 \text{ to } +21.50]$

$\text{SEPD}(X-Y) \ \& \ \text{SEPD}(X-XX) = 0.91 \times 25 = 22.75$
 $\text{SEPD}(Y-XX) = 0.62 \times 25 = 15.50$

68% Confidence Ranges

$\text{EBV}(X) - \text{EBV}(Y): [0 - 22.75] \text{ to } [0 + 22.75]$
 Chance $\text{BV}(X) > \text{BV}(Y)$ by 22.75 is **16%**
 $\text{EBV}(X) - \text{EBV}(XX): [6 - 22.75] \text{ to } [6 + 22.75]$
 Chance $\text{BV}(X) > \text{BV}(XX)$ by 28.75 is **16%**
 $\text{EBV}(Y) - \text{EBV}(XX): [6 - 15.50] \text{ to } [6 + 15.50]$
 Chance $\text{BV}(Y) > \text{BV}(XX)$ by 21.50 is **16%**
But, what could go up, could also go down!

G-BLUP and Risk

Genotyped sires will have ‘similar’ accuracy, SEP, SEPD ?
 unless many phenotyped progeny

Top 214, most reliability of 72-76%
 with no progeny

One with 3800 daughters, REL = 99%

<u>Accuracy</u>	<u>Reliability</u>	<u>BIF Accuracy</u>
0.30	0.09	0.05
0.40	0.16	0.08
0.50	0.25	0.13
0.60	0.36	0.20
0.70	0.49	0.28
0.80	0.64	0.40 G alone,
0.90	0.81	0.56 p ~ 50
0.99	0.98	0.86

G-BLUP and Risk
 Genotyped sires will have 'similar' accuracy, SEP, SEPD ?
But not same confidence ranges
CR will be centered on EBV
 Accuracy may depend on 'chip'
 Will covariance(PE_x, PE_y) ~ 0.00 ?

Bayesian 'Solutions'
 Accuracy and SEP cannot be obtained directly for current EBV solutions because coefficient matrix is too large to invert.
 Accuracy must be approximated and from approximated accuracy, approximate SEP. Instead use Bayes-Gibb's MCMC chains?

Bayesian 'Solutions'
 Gibb's MCMC chains provide empirical measures of prediction error variances
 Empirical PEV can be used to calculate approximate accuracies?
 Odds ratios?
 What about Covariance(PE_x, PE_y) ?
 From all pairs of chains ? [too many ?]
 1,000,000 chains, ~ 1,000,000,000,000/2 pairs

SUMMARY
 After 30 years, do we need to go beyond accuracy, reliability or BIF accuracy?
 Standard error of prediction uses accuracy and is on scale of units of measurement.
 Confidence ranges based on standard errors of prediction are quantitative measures of risk.

SUMMARY
 Standard error of prediction of difference between pairs of EBV uses accuracy of both EBV and is on scale of unit of measurement.
 Confidence range based on standard error of prediction of difference between EBV is measure of risk between two animals.
But, is difficult to explain and interpret!!

SUMMARY

Genomic BLUP may lead to accuracy being similar for many genotyped animals. Confidence ranges would differ only by EBV.

Bayesian analyses provide 'confidence ranges' directly without approximations and can be used to approximate accuracies.

THANK YOU
and
GOOD LUCK

Selection in simplest form is comparison of two animals: predict difference with measure of risk. The general "t-test"

$$\frac{[X_1 - X_2]}{\text{SQRT}[V(X_1) + V(X_2) - 2\text{COV}(X_1, X_2)]}$$

Similar for difference between two EBV
EBV(X) - EBV(Y)

Now prediction error for the difference is:
PED = [(EBV_x - true BV_x) - [(EBV_y - true BV_y)]